Recovering information in data exchange

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Data exchange/integration are crucial for achieving interoperability of applications.

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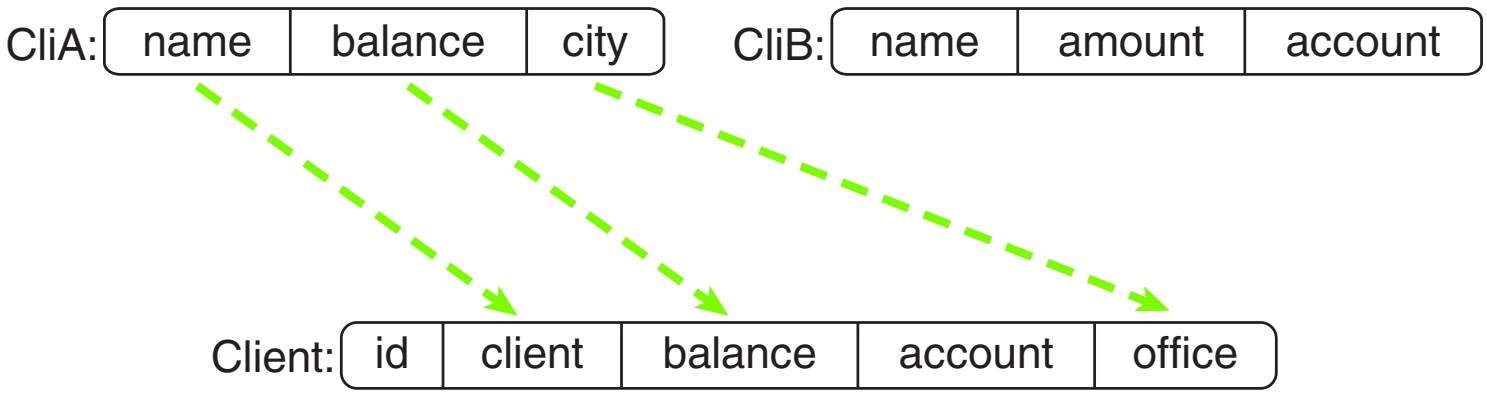

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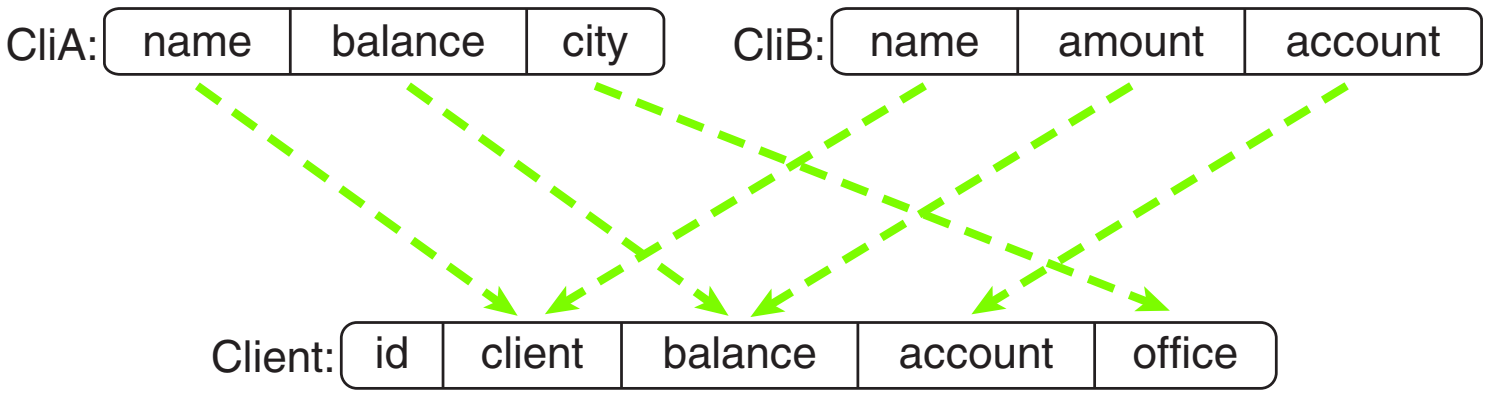

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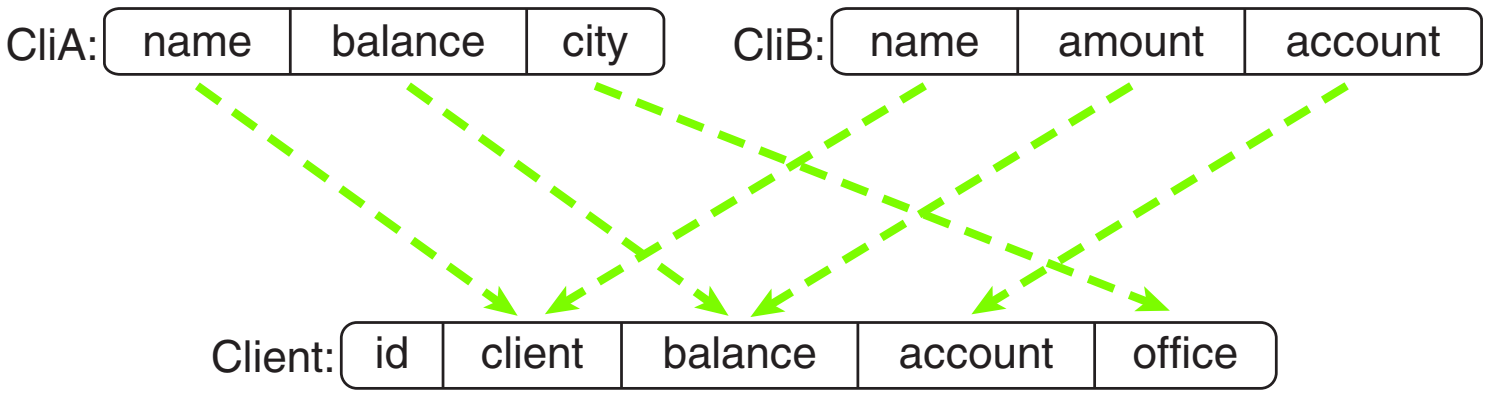

Schema mappings contain metadata.

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$\mathbf{S}_A$ $\mathbf{S}_B$ $\mathbf{S}_C$

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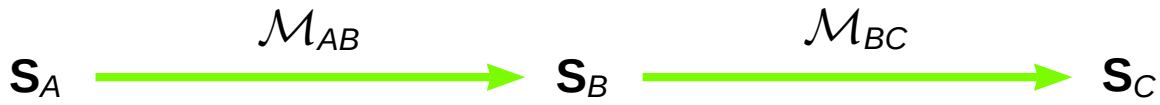

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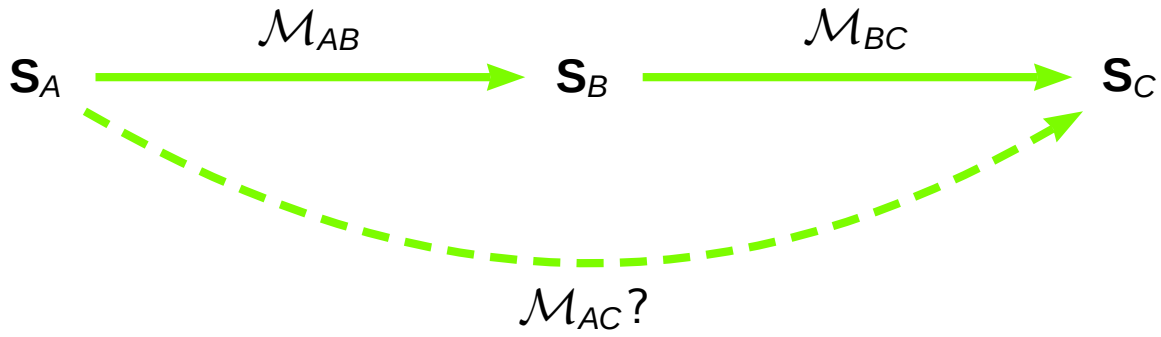

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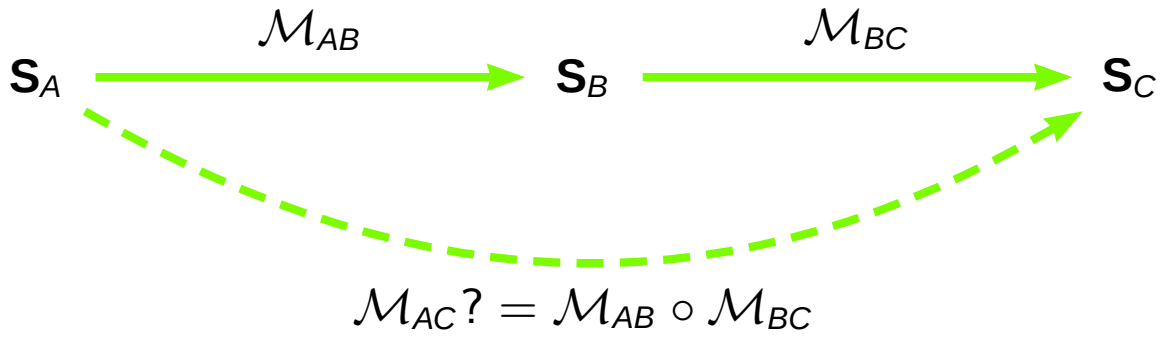

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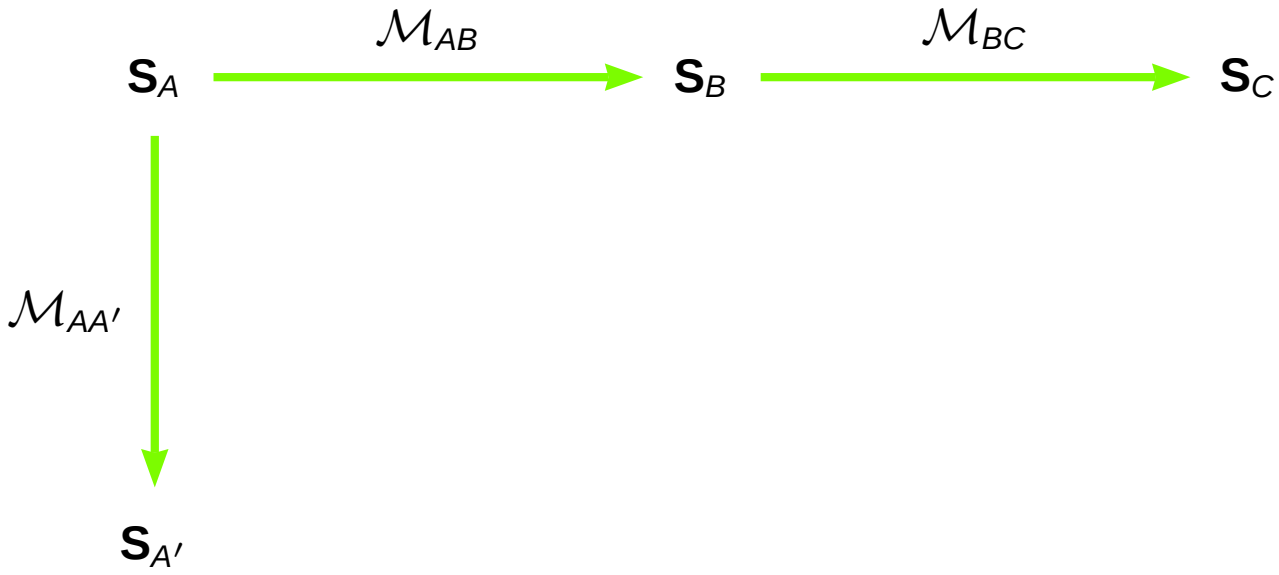

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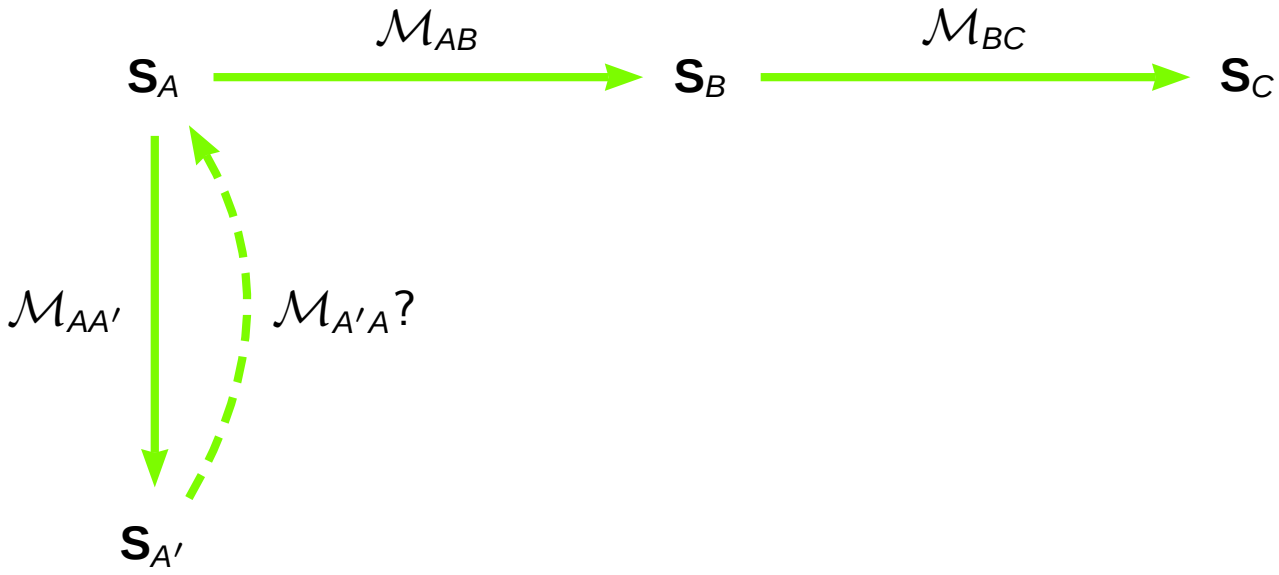

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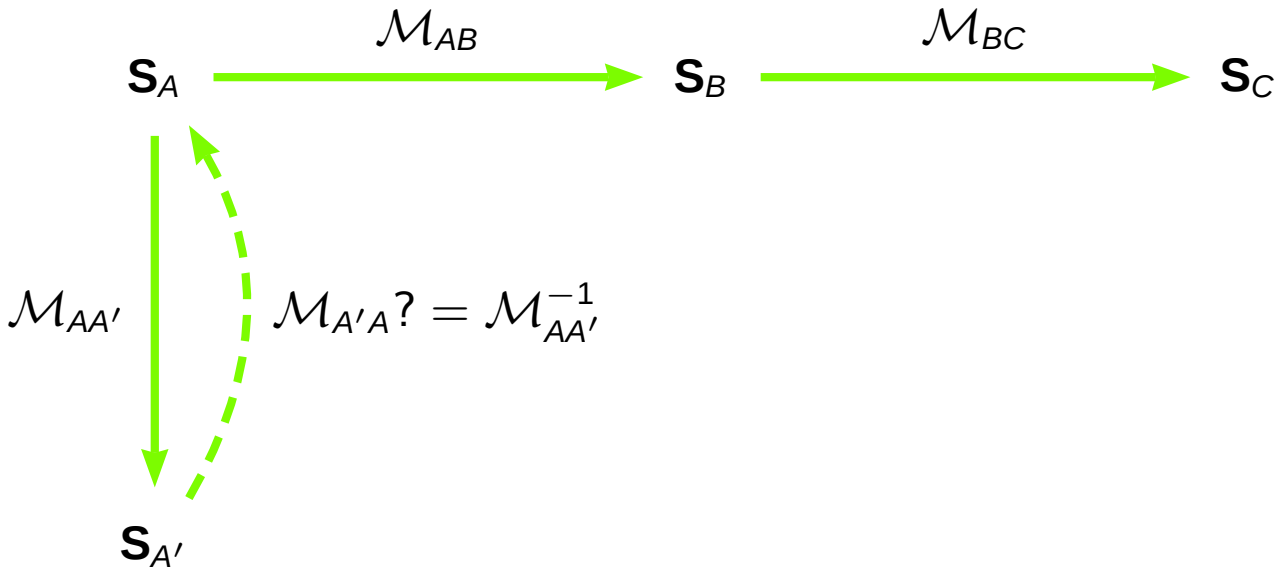

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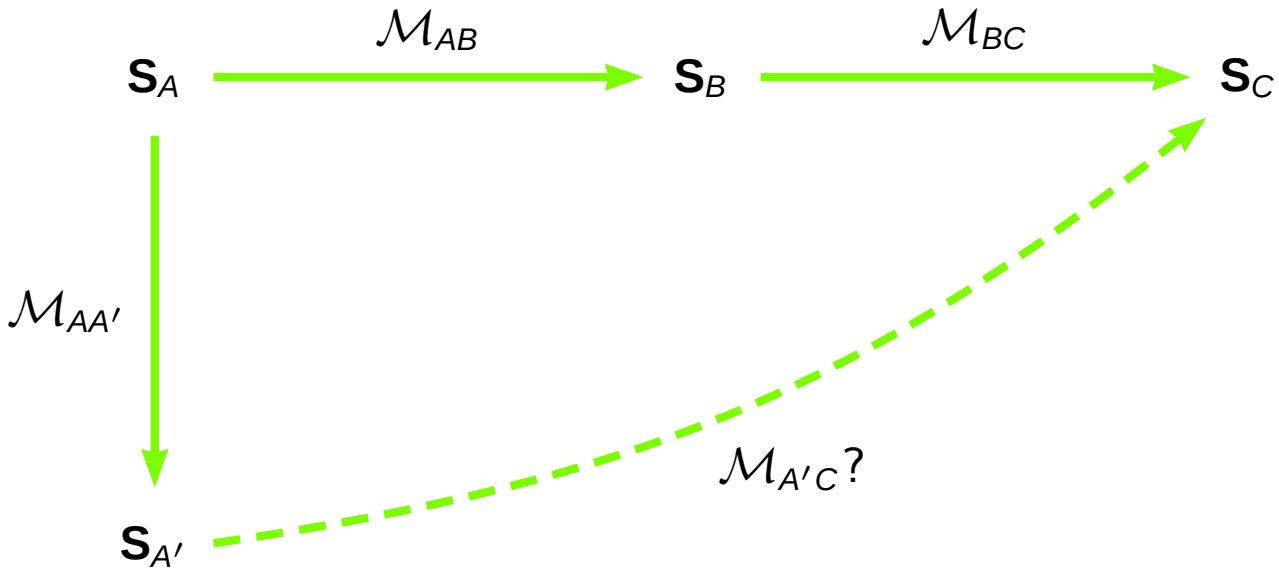

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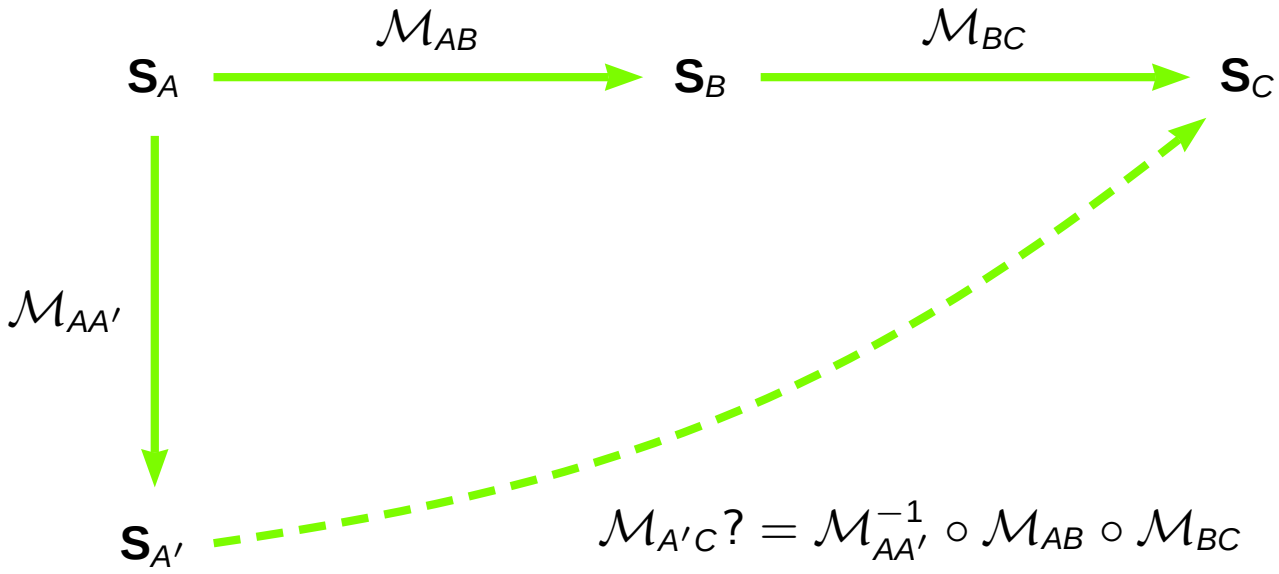

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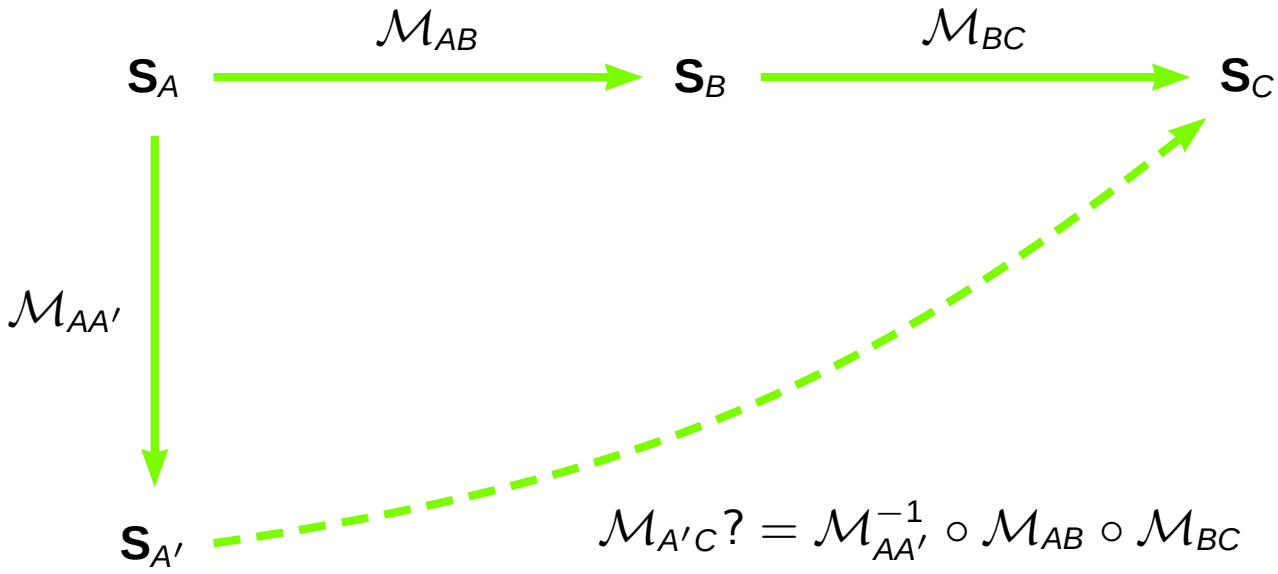

"The goal is to develop a model management engine that can support tools for all of these applications."

Phil Bernstein, Microsoft Research

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If $(I, J) \in \mathcal{M}$ then J is a *solution* for I under $\mathcal{M}$

- $J \in \mathrm{Sol}_{\mathcal{M}}(I)$.

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- Source-to-target tuple-generating dependencies (st-tgds):

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- A set Σ of dependencies *specifies* $\mathcal{M}$:

$$(I, J) \in \mathcal{M} \iff (I, J) \models \Sigma.$$

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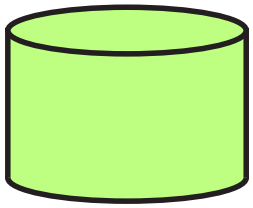

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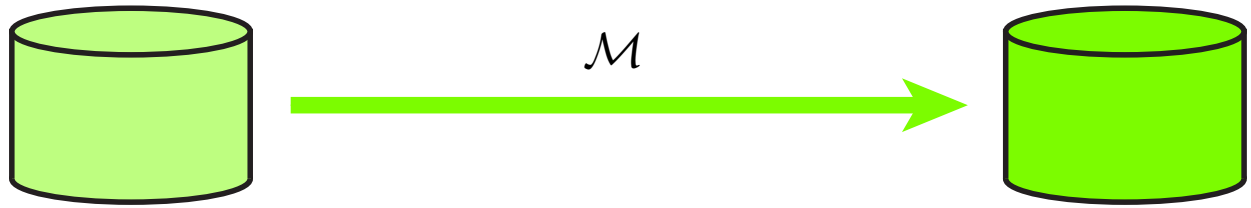

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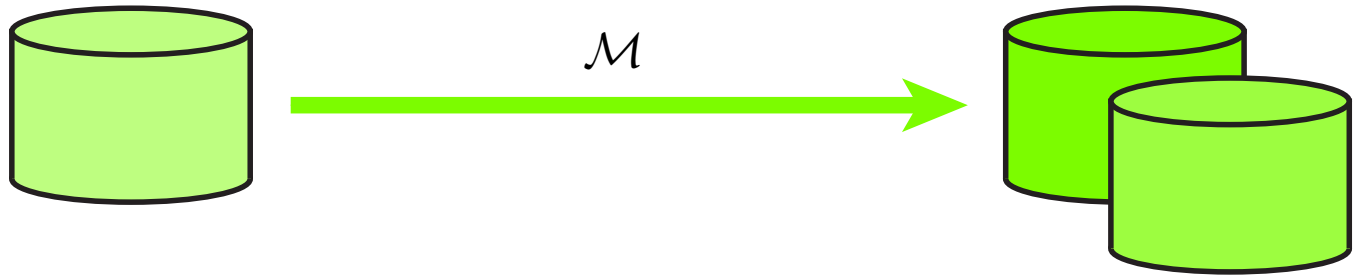

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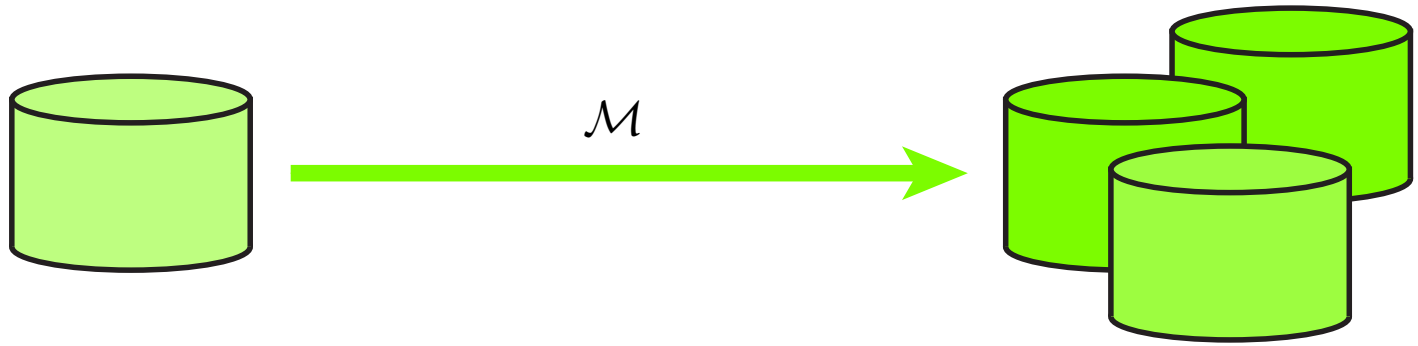

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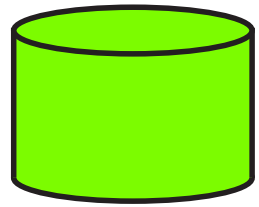

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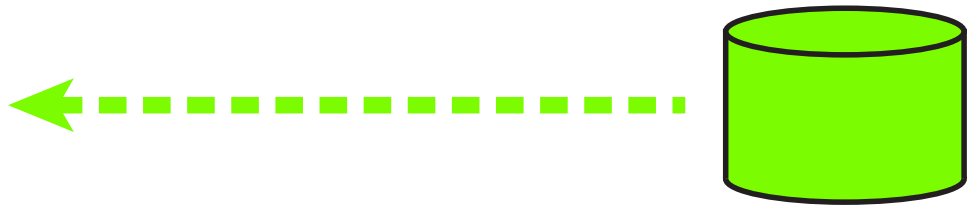

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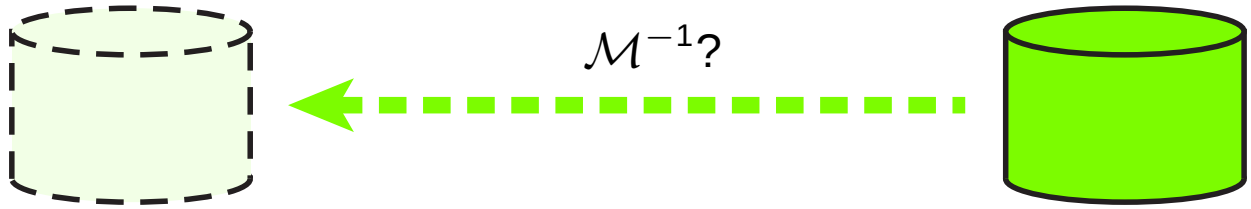

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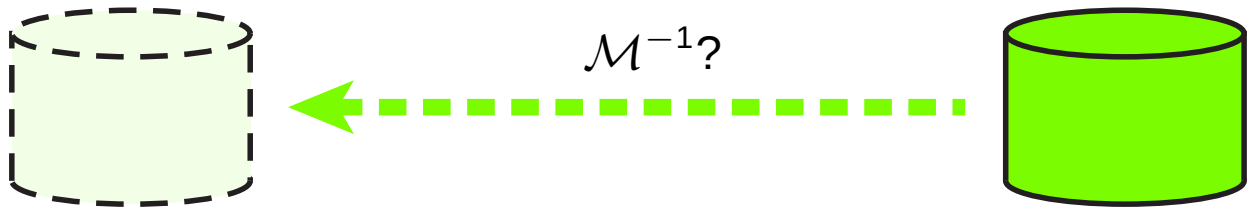

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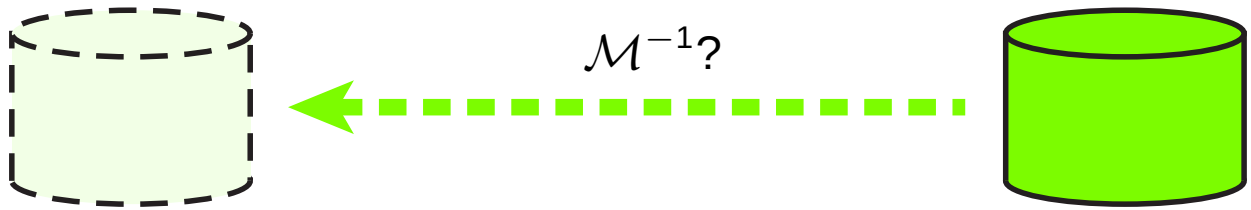

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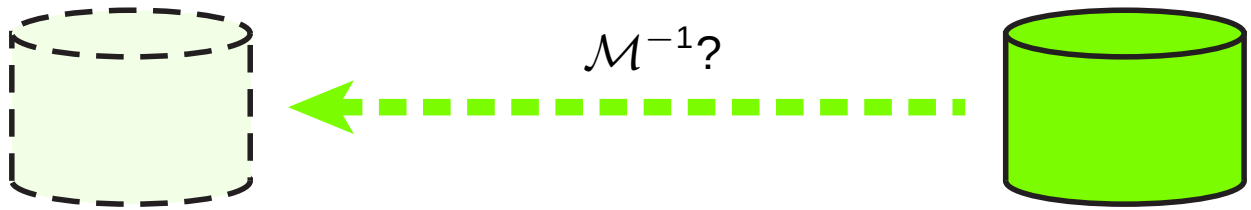

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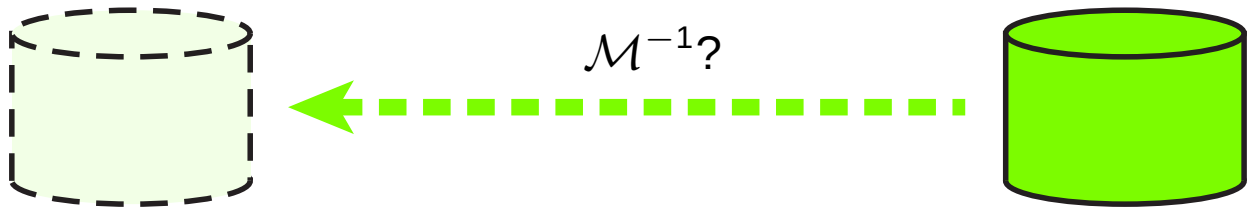

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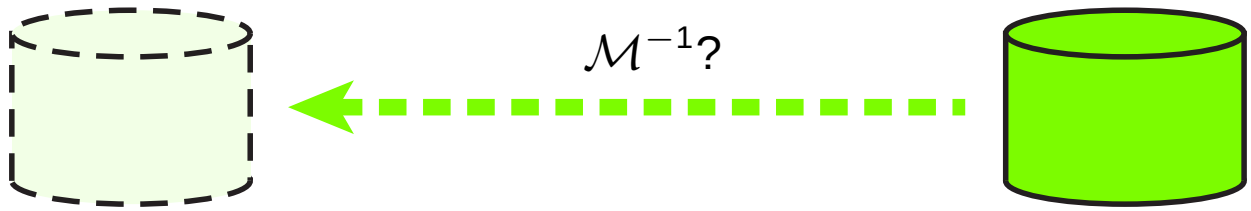

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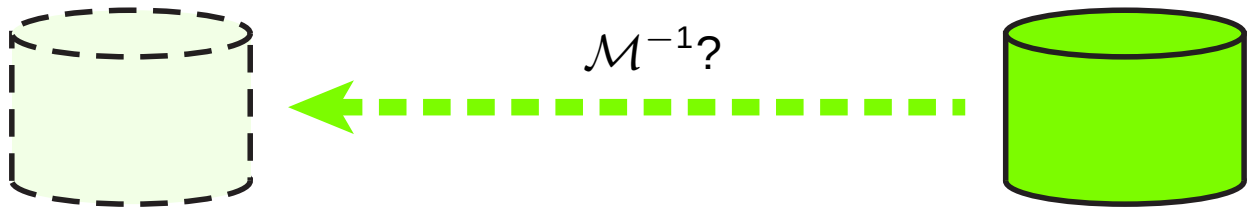

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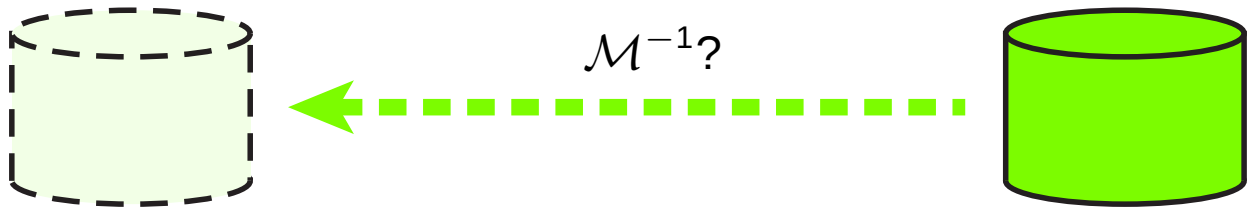

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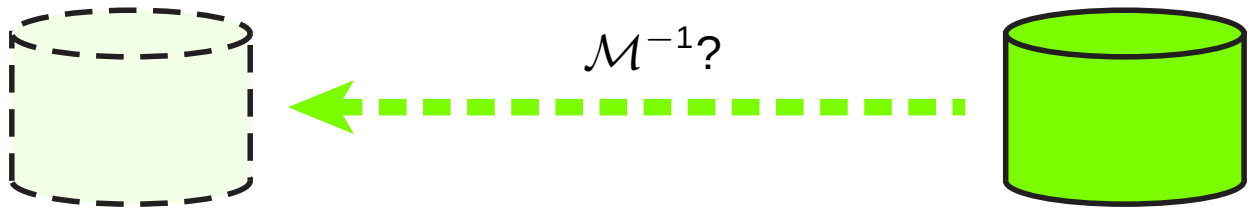

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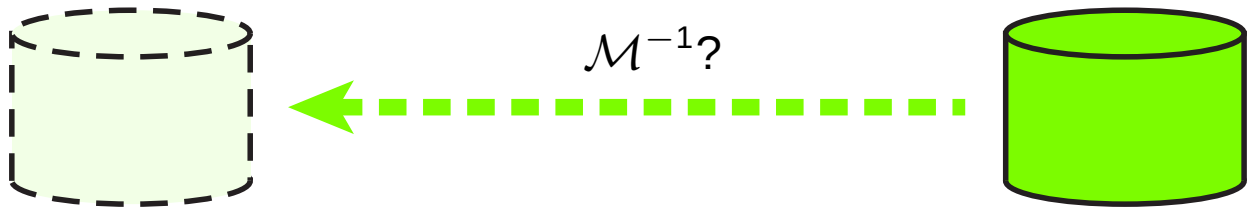

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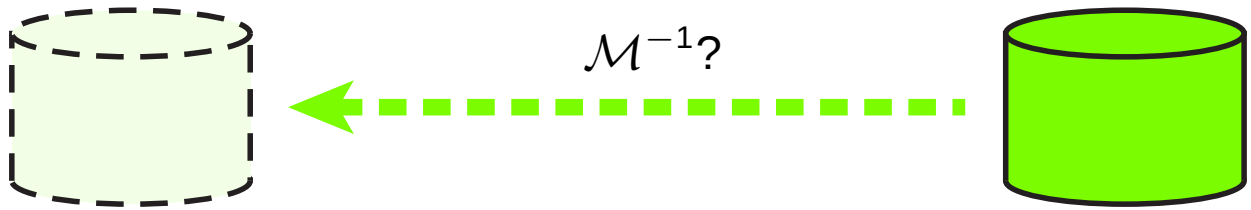

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What is the mapping from B to A ?

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- we can recover the exchanged data (or part of it).
- we can specify it in the same language (or slightly different).
- we can always compute it.

How can we give a consistent semantics to the notion of inverse?

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$\mathcal{M}_1$: $\quad \mathsf{B}(U, x, y) \quad \longrightarrow \quad \exists Y_1 \; \exists Z_1 \; \exists Z_2 \quad \mathsf{A}(x, Y_1, Z_1, Z_2)$

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what means to recover sound information

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We call mapping $\mathcal{M}_1$ and $\mathcal{M}_2$ *recoveries* of $\mathcal{M}$.

A mapping is a *recovery* of $\mathcal{M}$ if

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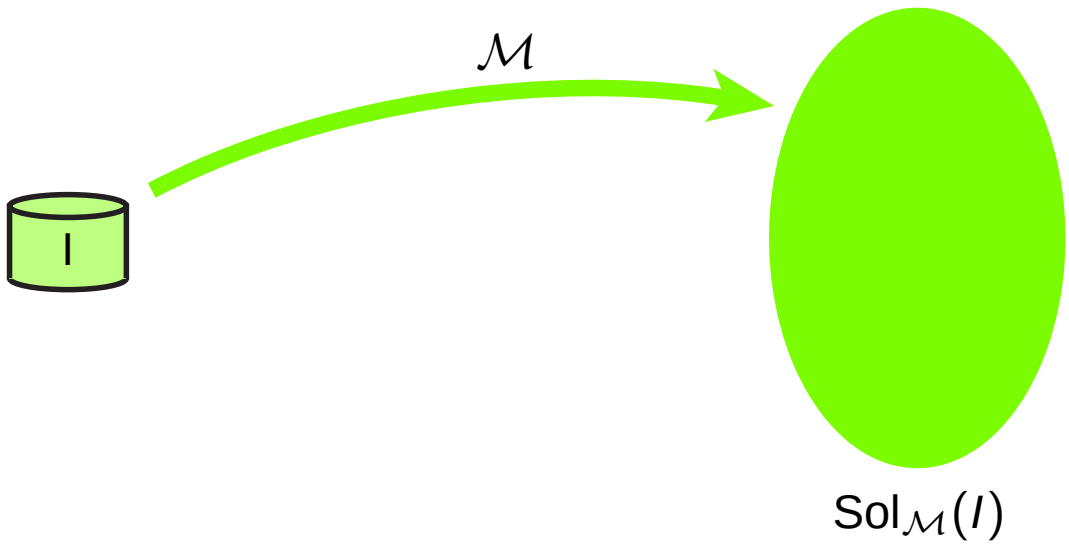

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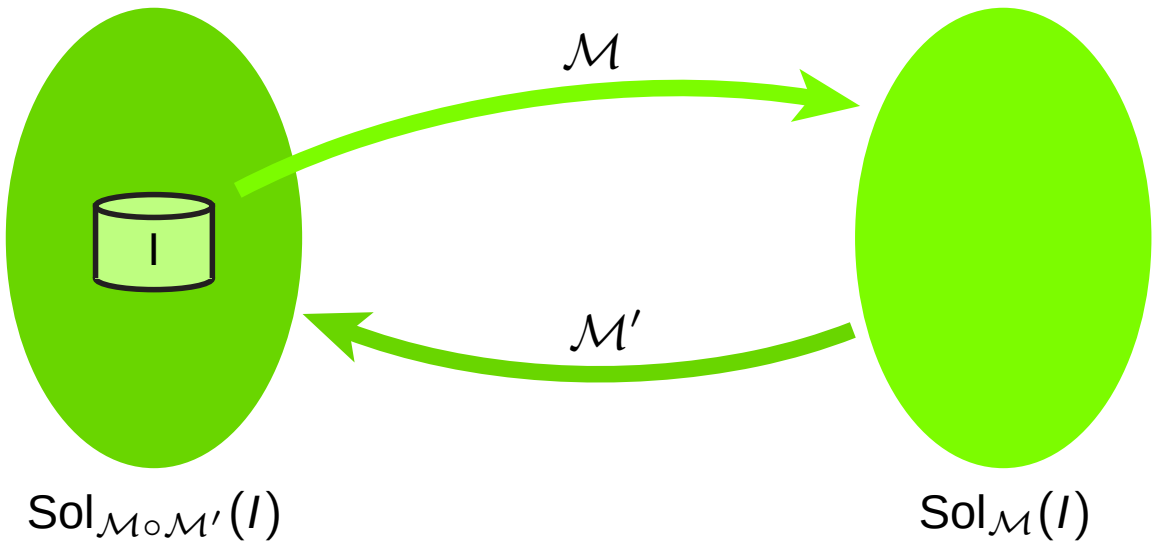

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$\mathcal{M}_4$ *is better than* $\mathcal{M}_2$ and $\mathcal{M}_1$

We call mapping $\mathcal{M}_4$ a *maximum recovery* of $\mathcal{M}$.

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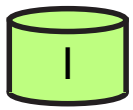

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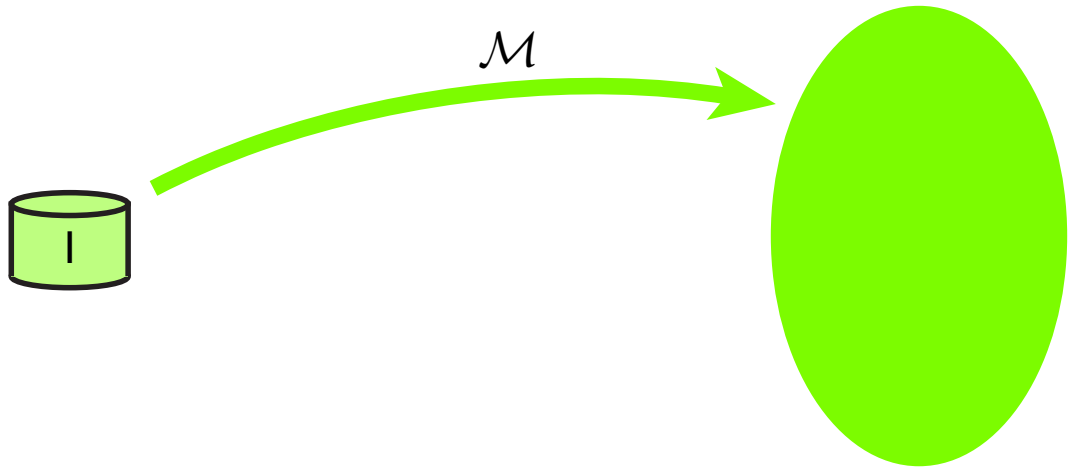

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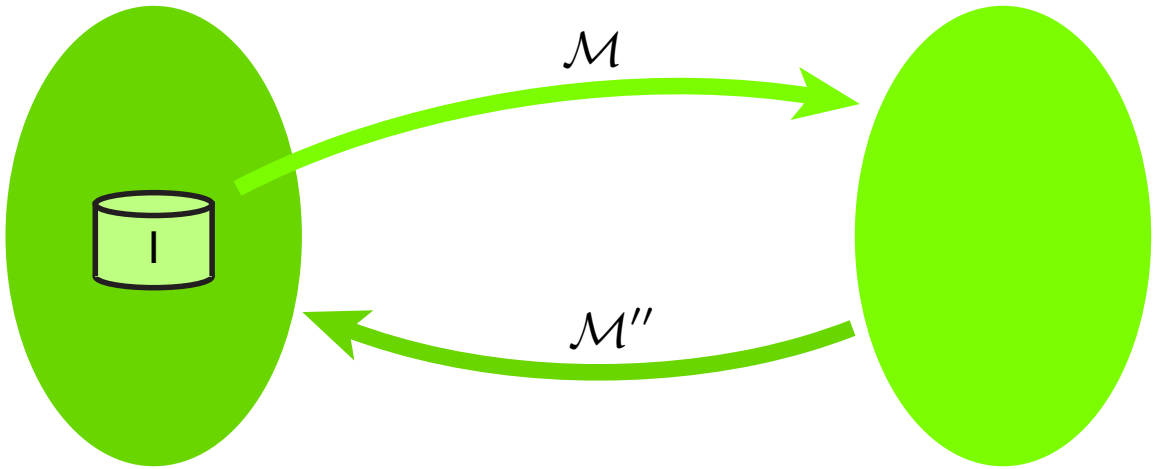

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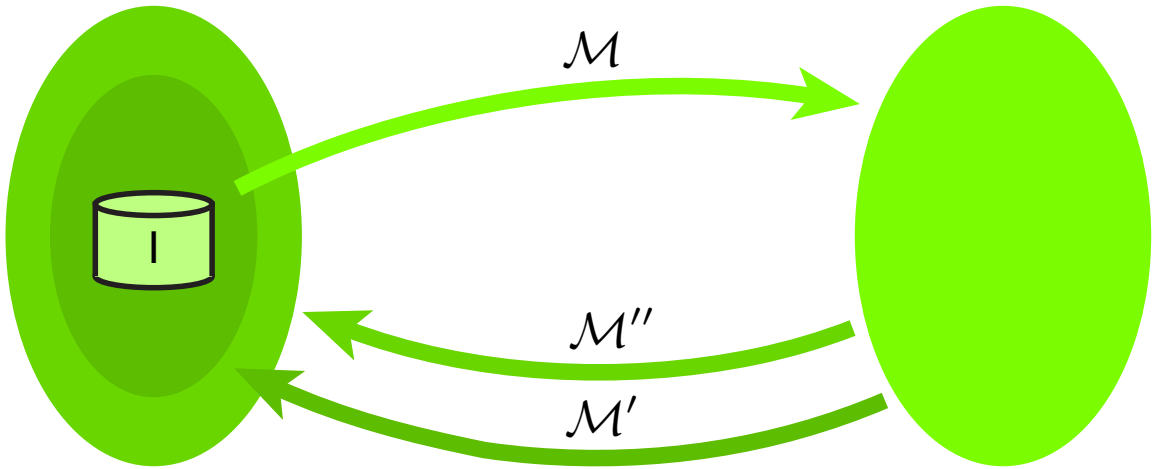

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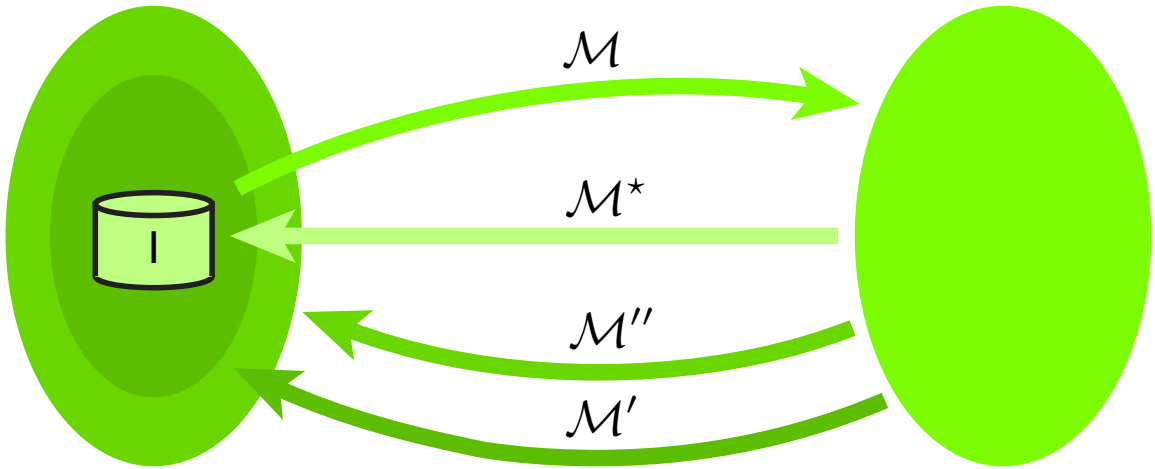

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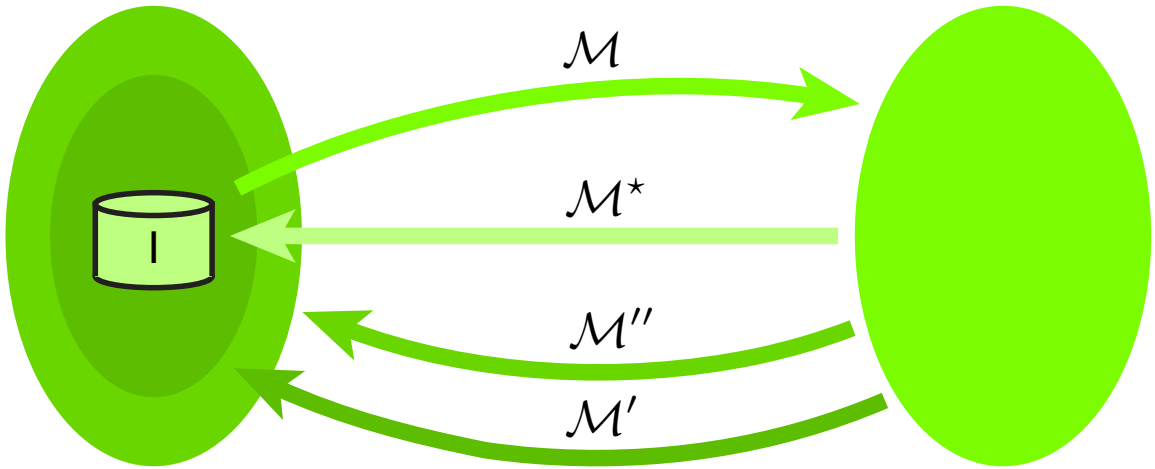

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$\mathcal{M}'$ is a *maximum recovery* of $\mathcal{M}$ if $\mathcal{M}'$ is a maximum w.r.t. $\preceq$.

Maximum recovery strictly generalize previous notions

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Let $\mathcal{M}$ be a mappings specified by st-tgds and fagin-invertible, then:

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Advantage of maximum recoveries:

- Fagin inverses rarely exist for st-tgds.
- Maximum recoveries always exist for st-tgds.

How can we compute maximum recoveries?

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If $\mathcal{M}$ is specified by st-tgds, then for every target query $Q \in \mathbf{CQ}$, there is a source rewriting $Q' \in \mathbf{UCQ}^{=}$ of Q under $\mathcal{M}$.

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For every dependency $\varphi(\bar{x}) \rightarrow \exists \bar{y}\ \psi(\bar{x}, \bar{y})$ defining $\mathcal{M}$:

- Let $\alpha(\bar{x}) \in \mathbf{UCQ}^{=}$ be a source rewriting of $\exists \bar{y} \psi(\bar{x}, \bar{y})$ under $\mathcal{M}$.
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Computing a maximum recovery using rewriting of queries

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	full st-tgds	st-tgds
Output:	**CQ-to-UCQ**$^{=}$	**CQ**$^{\mathbf{C}(\cdot)}$**-to-UCQ**$^{=}$
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Some highlights:

- We use query-rewriting as a black box in the algorithm.

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Some highlights:

- We use query-rewriting as a black box in the algorithm.
- Disjunction and predicate $\mathbf{C}(\cdot)$ is the language needed to specify maximum recoveries for st-tgds.

We need to extend the language of st-tgds in order to express maximum recoveries

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- The algorithm returns a set of **CQ**$^{\mathbf{C}(\cdot)}$**-to-UCQ**$^{=}$ dependencies.

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We would like a natural notion of inverse such that:

- st-tgds always have an inverse, and
- such inverse can be expressed in a language with the same good properties as st-tgds for data exchange.

How can we compute an inverse without disjunctions?

We can parameterize the notion of maximum recovery by a class of queries

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If we concentrate in conjunctive information as st-tgds, we can find more practical inverses.

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Example

$$\mathcal{M}: \quad A(x,y,z) \longrightarrow \exists U\, B(x,y,U)$$

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We call mapping $\mathcal{M}'$ a Q_1-*recovery* of $\mathcal{M}$.

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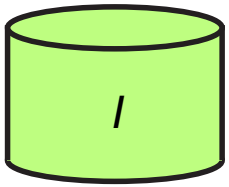

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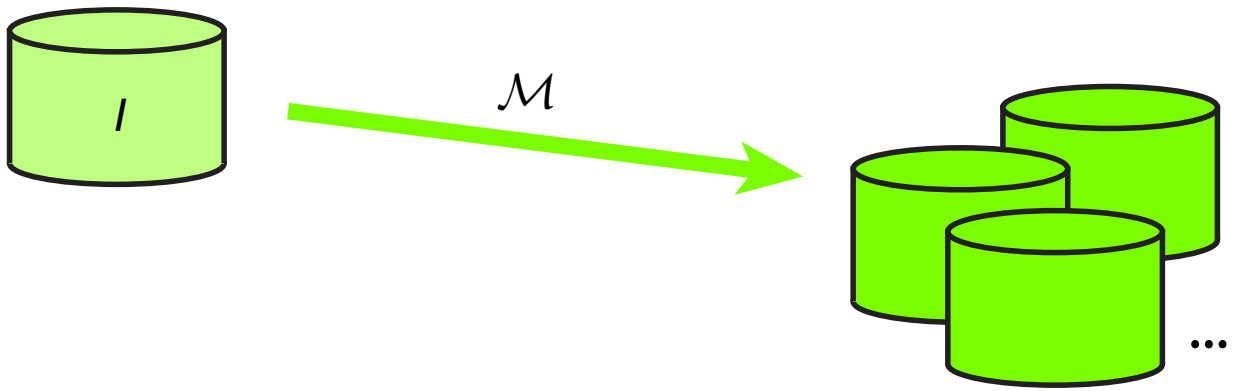

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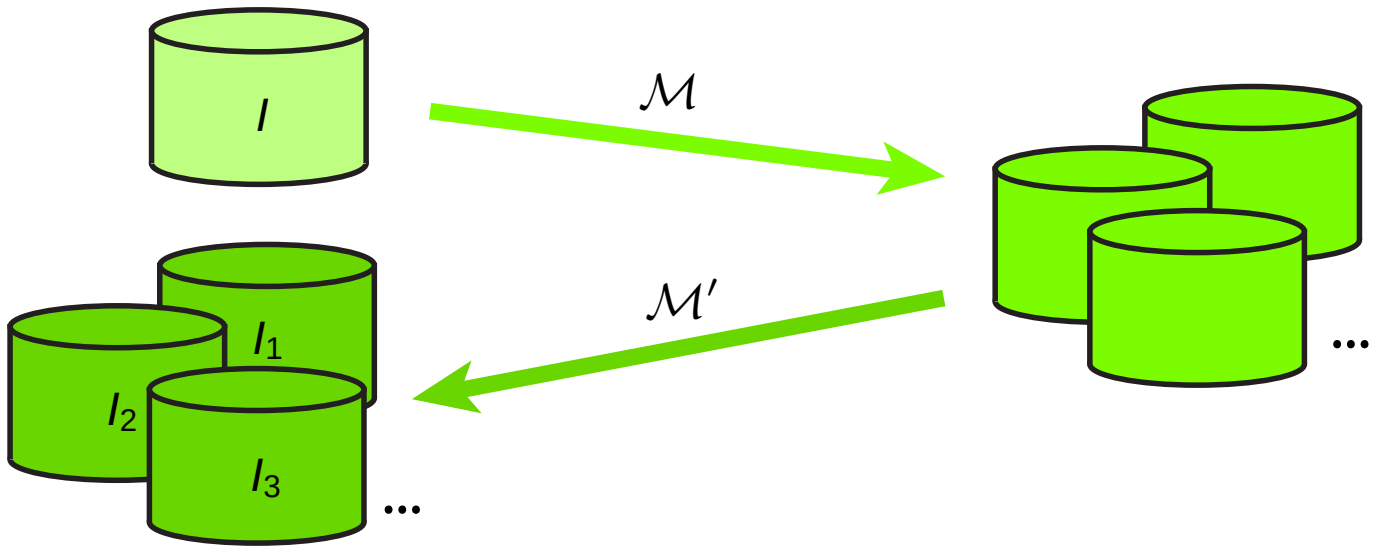

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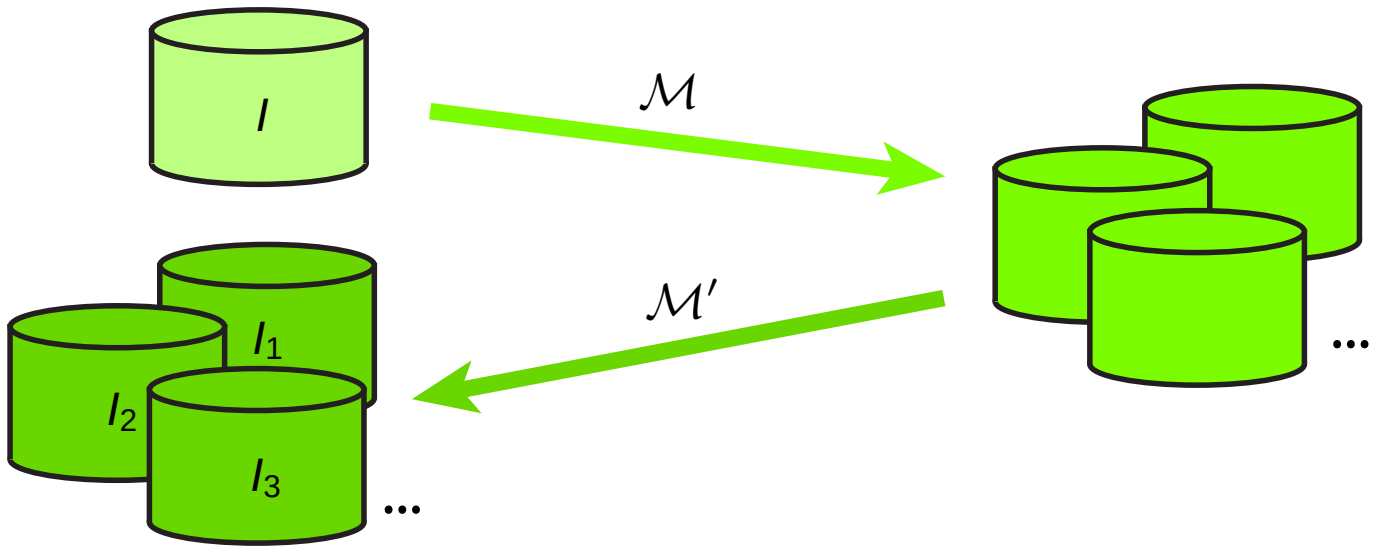

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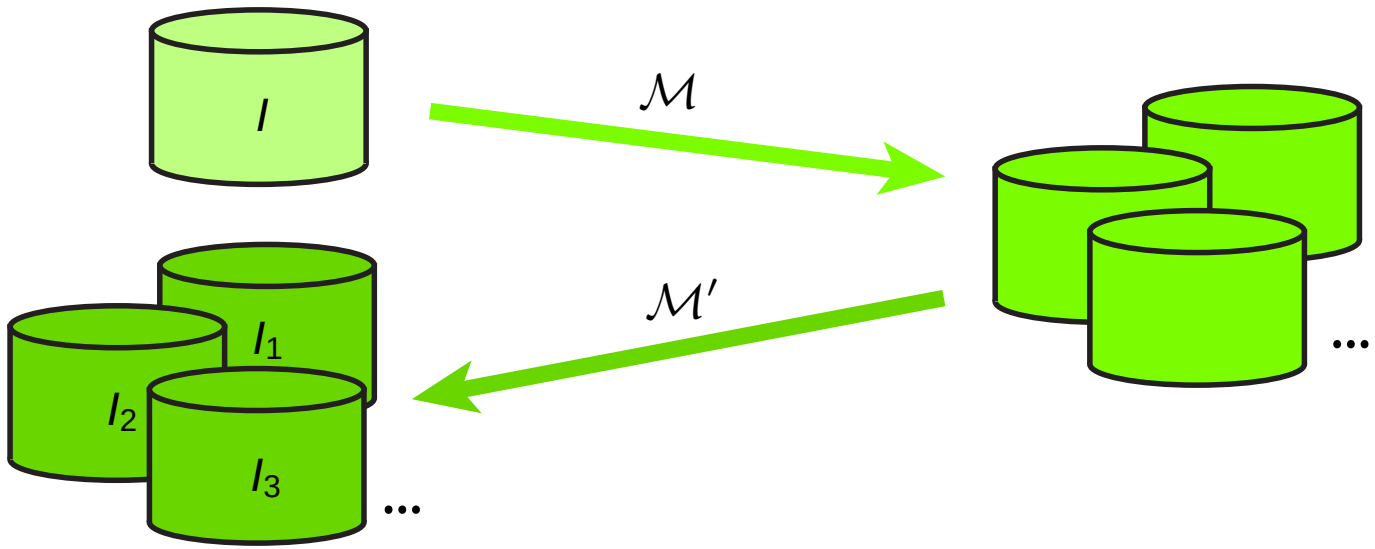

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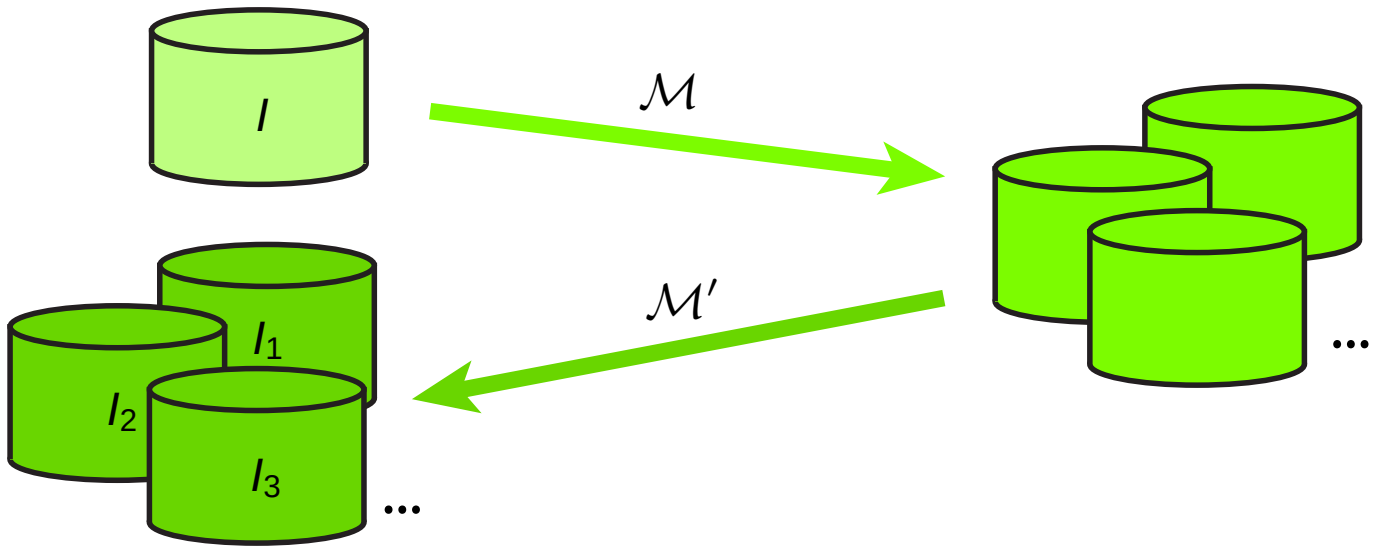

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We want a mapping such that the certain answers are *as close as possible* to $Q(I)$.

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$\mathcal{M}_1$ is better that any other possible $\mathcal{C}$-recovery!

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Theorem

Every mapping specified by st-tgds has a **CQ**-maximum recovery that can be specified by st-tgds with $\neq$ and $\mathbf{C}(\cdot)$ in the left-hand side.

How can we compute a **CQ**-maximum recovery?

Computing a **CQ**-maximum recovery consists of two main steps

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Algorithm

- Step 1: Compute a maximum recovery $\mathcal{M}'$ of $\mathcal{M}$.
- Step 2: Eliminate disjunctions and equalities of $\mathcal{M}'$.
  - Step 2.1: Eliminate equalities.
  - Step 2.2: Eliminate disjunctions.

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Problem: disjunctions and equalities in the right-hand side.

Step 2.1: Eliminate right-hand side equalities

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- Let $\mathcal{M}'$ be the mapping constructed in Step 1.
- Construct $\mathcal{M}''$ from $\mathcal{M}'$ by replacing right-hand side equalities by inequalities in the left-hand side.

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Step 2.1: Eliminate right-hand side equalities

Example

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The key concepts in Step 2.2 are
homomorphisms and *Cartesian product of queries*.

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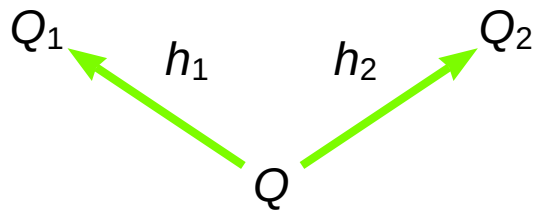

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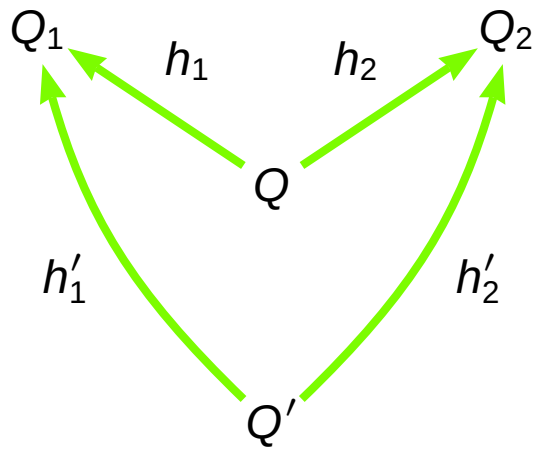

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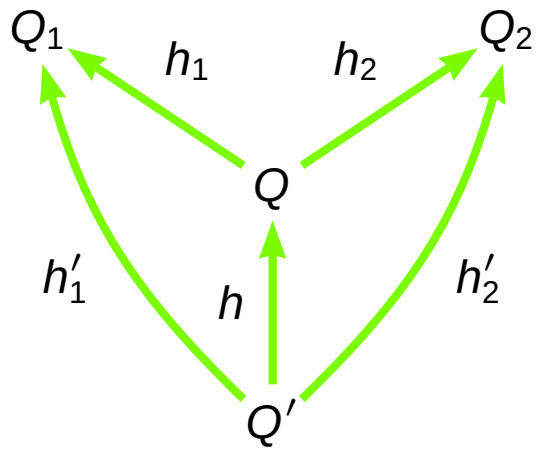

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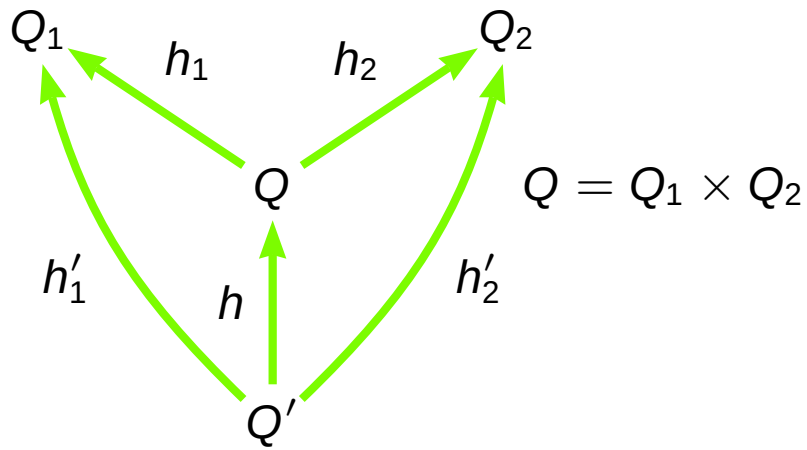

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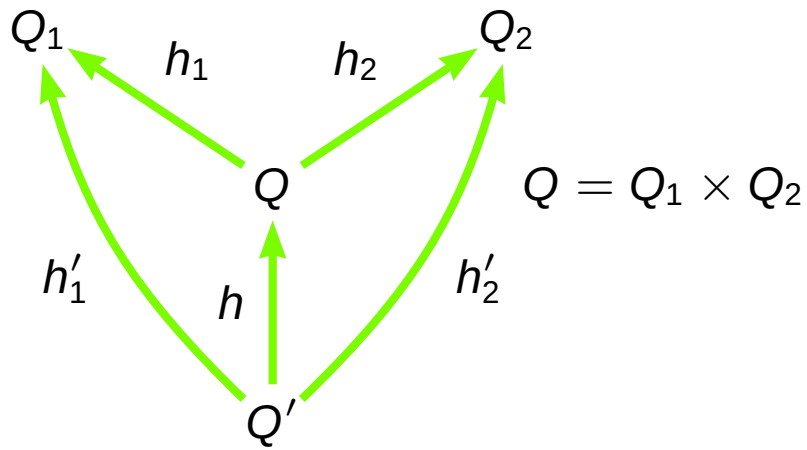

A simple extension of the Cartesian product of graphs.

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The output is a **CQ**-maximum recovery of $\mathcal{M}$
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- If we focus in certain kind of queries we can find more practical solutions to *recovering information in data exchange*.

Recovering information in data exchange

Cristian Riveros
Khipu Institute

Oxford University
Tue 26 Jan 2010