Recovering information in data exchange

Cristian Riveros
Khipu Institute

Oxford University
Tue 26 Jan 2010
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- 40% of the effort in a software project is spent in mapping similar data.
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- 50% of total budget in IT projects are spent on integrating applications.

- 40% of the effort in a software project is spent in mapping similar data.

Data exchange/integration are crucial for achieving interoperability of applications.
Schema mappings are essential to perform the exchange of data.
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Schema mapping describes the relationship between schemas.
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**CliA:**
- name
- balance
- city

**CliB:**
- name
- amount
- account
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Schema mapping describes the relationship between schemas.

**CliA:**
- name
- balance
- city

**CliB:**
- name
- amount
- account

**Client:**
- id
- client
- balance
- account
- office
Schema mappings are essential to perform the exchange of data.

Schema mapping describes the relationship between schemas.

CliA: name | balance | city

CliB: name | amount | account

Client: id | client | balance | account | office
Schema mappings are essential to perform the exchange of data.

Schema mapping describes the relationship between schemas.

CliA: name  balance  city

CliB: name  amount  account

Client: id  client  balance  account  office
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Schema mapping describes the relationship between schemas.

```
CliA:  name  balance  city
CliB:  name  amount  account

Client:  id  client  balance  account  office
```

Schema mappings contain metadata.
In several applications we need to reuse the metadata of schema mappings
In several applications we need to reuse the metadata of schema mappings $S_A$, $S_B$, $S_C$. 
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\[ M_{AC} = M_{AB} \circ M_{BC} \]
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\[ M_{AA'} \] 

\[ M_{A'A} = M_{AA'}^{-1} \]
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\[ \mathcal{M}_{A'B} = \mathcal{M}_{AA'}^{-1} \circ \mathcal{M}_{AB} \circ \mathcal{M}_{BC} \]
In several applications we need to reuse the metadata of schema mappings

\[ \mathcal{S}_A \xrightarrow{\mathcal{M}_{AB}} \mathcal{S}_B \xrightarrow{\mathcal{M}_{BC}} \mathcal{S}_C \]

\[ \mathcal{M}_{A'} = \mathcal{M}_{AA'}^{-1} \circ \mathcal{M}_{AB} \circ \mathcal{M}_{BC} \]

“The goal is to develop a model management engine that can support tools for all of these applications.”

Phil Bernstein, Microsoft Research
In recent years, there has been an increasing interest to develop solid foundations for metadata management.
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2003  *Applying model management to classical meta data problems.*
       P. Bernstein. *CIDR.*

2005  *Composing schema mappings: second-order dependencies* ...
       R. Fagin, P. G. Kolaitis, L. Popa, and W.-C. Tan. *PODS.*

2006  *Inverting schema mappings.*
       R. Fagin. *PODS.*

2007  *Quasi-inverses of schema mappings*
       R. Fagin, P. G. Kolaitis, L. Popa, and W.-C. Tan. *PODS.*

2008  *The recovery of a schema mapping: bringing exchanged* ...
       M. Arenas, J. Pérez, and C. Riveros. *PODS.*

2009  *Inverting schema mappings: bridging the gap* ...
       M. Arenas, J. Pérez, J. Reutter, and C. Riveros. *VLDB.*
In recent years, there has been an increasing interest to develop solid foundations for metadata management

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A new semantics on how to invert a schema mapping
A new semantics on how to invert a schema mapping

*Maximum recovery*

- A notion of inverse based on recovering sound information.
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*\(C\)-maximum recovery for a class of queries \(C\)*
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C-maximum recovery for a class of queries C
- A parameterized notion of maximum recovery.
A new semantics on how to invert a schema mapping

**Maximum recovery**
- A notion of inverse based on recovering sound information.
- An algorithm to compute maximum recoveries.

**C-maximum recovery for a class of queries C**
- A parameterized notion of maximum recovery.
- An algorithm to compute CQ-maximum recoveries.
Outline
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Schema mappings are the building blocks to perform data exchange.
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Two database schemas: \(S\) (source) and \(T\) (target).
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Two database schemas: $\mathbf{S}$ (source) and $\mathbf{T}$ (target).

A mapping $\mathcal{M}$ is a set of pairs $(I, J)$ with:

- $I$ a source instance,
- $J$ a target instance.
Schema mappings are the building blocks to perform data exchange.

Two database schemas: $S$ (source) and $T$ (target).

A mapping $M$ is a set of pairs $(I, J)$ with:
- $I$ a source instance,
- $J$ a target instance.

If $(I, J) \in M$ then $J$ is a solution for $I$ under $M$:
- $J \in \text{Sol}_M(I)$. 
Schema mappings are usually given in the form of logical specifications
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- Source-to-target tuple-generating dependencies (st-tgds):

\[ \varphi_S(\bar{x}) \rightarrow \psi_T(\bar{x}) \]

with \( \varphi_S(\bar{x}) \) and \( \psi_T(\bar{x}) \) a CQ-query.
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**Example**

Source: \{ClientA(name, balance, city) \}
Target: \{ClientB(id, name, balance, account) \}
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Example

Source: \( \{ \text{ClientA(name, balance, city)} \} \)

Target: \( \{ \text{ClientB(id, name, balance, account)} \} \)

\[ \exists Z \text{ClientA}(x, y, Z) \rightarrow \exists U \exists V \text{ClientB}(U, x, y, V) \]
Schema mappings are usually given in the form of logical specifications

- Source-to-target tuple-generating dependencies (st-tgd s):

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Example

Source:  \{ClientA(name, balance, city) \}
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**Example**

Source: \( \{ \text{ClientA(name, balance, city) } \} \)

Target: \( \{ \text{ClientB(id, name, balance, account) } \} \)

\[
\text{ClientA}(x, y, Z) \rightarrow \exists U \exists V \text{ClientB}(U, x, y, V)
\]

- A set \( \Sigma \) of dependencies specifies \( M \):

\[
(l, J) \in M \iff (l, J) \models \Sigma.
\]
Our goal is to find the semantics of the notion of inverse
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Our goal is to find the semantics of the notion of inverse $M^{-1}$?
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Example

\[ \mathcal{M}: A(x, y) \rightarrow \exists Z B(x, Z) \]
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Example

\[ M: \quad A(x, y) \rightarrow \exists Z \ B(x, Z) \]

\[ I: \quad \{ A(1, 1) \} \]
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Example

\( \mathcal{M}: \quad A(x, y) \rightarrow \exists Z \ B(x, Z) \)

\( l: \quad \{ A(1, 1) \} \)

\( \text{Sol}_{\mathcal{M}}(l): \quad \{ B(1, 2) \} \)
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\[ \mathcal{M}: \quad A(x, y) \rightarrow \exists Z B(x, Z) \]

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\[
\text{Sol}_{\mathcal{M}}(I): \quad \{ B(1, 2) \}, \{ B(1, 1), B(2, 3) \}
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Example

\[ \mathcal{M}: \quad A(x, y) \rightarrow \exists Z \ B(x, Z) \]

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\[ \text{Sol}_{\mathcal{M}}(l): \quad \{ B(1, 2) \}, \{ B(1, 1), B(2, 3) \}, \ldots \]
Our goal is to find the semantics of the notion of inverse $M^{-1}$?

Example

$M$: $A(x, y) \rightarrow \exists Z B(x, Z)$

$I$: \{ $A(1, 1)$ \}

$\text{Sol}_M(I)$: \{ $B(1, 2)$ \}, \{ $B(1, 1), B(2, 3)$ \}, ... 

$J$: \{ $B(1, 2)$ \}
Our goal is to find the semantics of the notion of inverse

Example

\[ \mathcal{M}: \quad A(x, y) \rightarrow \exists Z \ B(x, Z) \]

\[ I: \quad \{ A(1, 1) \} \]

\[ \text{Sol}_\mathcal{M}(I): \quad \{ B(1, 2) \}, \{ B(1, 1), B(2, 3) \}, \ldots \]

\[ J: \quad \{ B(1, 2) \} \]

\[ \text{Sol}_{\mathcal{M}^{-1}}(J): \quad \{ A(1, 1) \} \]
Our goal is to find the semantics of the notion of inverse

Example

\[ M: \quad A(x, y) \rightarrow \exists Z \ B(x, Z) \]

\[ I: \quad \{ A(1, 1) \} \]

\[ \text{Sol}_M(I): \quad \{ B(1, 2) \} , \{ B(1, 1), B(2, 3) \} , \ldots \]

\[ J: \quad \{ B(1, 2) \} \]

\[ \text{Sol}_{M^{-1}}(J): \quad \{ A(1, 1) \} , \{ A(1, 2), A(1, 3) \} \]
Our goal is to find the semantics of the notion of inverse

Example

\[ M: \quad A(x, y) \rightarrow \exists Z \ B(x, Z) \]

\[ l: \quad \{ A(1, 1) \} \]

\[ Sol_M(l): \quad \{ B(1, 2) \}, \{ B(1, 1), B(2, 3) \}, \ldots \]

\[ j: \quad \{ B(1, 2) \} \]

\[ Sol_{M^{-1}}(l): \quad \{ A(1, 1) \}, \{ A(1, 2), A(1, 3) \}, \{ \emptyset \}, \ldots \]
Our goal is to find the semantics of the notion of inverse

Example

\[ M: \quad A(x, y) \rightarrow \exists Z \ B(x, Z) \]

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\[ \text{Sol}_M(I): \quad \{ B(1, 2) \}, \{ B(1, 1), B(2, 3) \}, \ldots \]

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What is the mapping from B to A?
We want a consistent and useful semantics of the notion of inverse.
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We want a semantics of the notion of inverse, such that:
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We want a semantics of the notion of inverse, such that:

- we can recover the exchanged data (or part of it).
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- we can recover the exchanged data (or part of it).

- we can specify it in the same language (or slightly different).
We want a consistent and useful semantics of the notion of inverse.

- We can recover the exchanged data (or part of it).
- We can specify it in the same language (or slightly different).
- We can always compute it.
How can we give a consistent semantics to the notion of inverse?
We want to define what means to recover sound information
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- data *may be lost* in the exchange through $\mathcal{M}$.
We want to define what means to recover sound information.

- data *may be lost* in the exchange through $\mathcal{M}$.
- we want an $\mathcal{M}'$ that *at least* recovers sound data w.r.t. $\mathcal{M}$. 
We want to define what means to recover sound information.

- Data may be lost in the exchange through \( \mathcal{M} \).
- We want an \( \mathcal{M}' \) that at least recovers sound data w.r.t. \( \mathcal{M} \).

**Example**

\[ \mathcal{M}: \quad A(x, y, Z, Z) \quad \rightarrow \quad \exists U \ B(U, x, y) \]
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**Example**

\[ \mathcal{M} : \quad A(x, y, Z, Z) \rightarrow \exists U \quad B(U, x, y) \]

\[ \mathcal{M}_1 : \quad B(U, x, y) \rightarrow \exists Y_1 \exists Z_1 \exists Z_2 \quad A(x, Y_1, Z_1, Z_2) \]
We want to define what means to recover sound information

- data *may be lost* in the exchange through $\mathcal{M}$.
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Example

$\mathcal{M}$: $A(x, y, Z, Z) \rightarrow \exists U \ B(U, x, y)$

$\mathcal{M}_1$: $B(U, x, y) \rightarrow \exists Y_1 \exists Z_1 \exists Z_2 \ A(x, Y_1, Z_1, Z_2)$ ✓
We want to define what means to recover sound information data may be lost in the exchange through \( \mathcal{M} \).

- we want an \( \mathcal{M}' \) that at least recovers sound data w.r.t. \( \mathcal{M} \).

Example

\[
\begin{align*}
\mathcal{M}: & \quad A(x, y, Z, Z) \quad \rightarrow \quad \exists U \ B (U, x, y) \\
\mathcal{M}_1: & \quad B(U, x, y) \quad \rightarrow \quad \exists Y_1 \exists Z_1 \exists Z_2 \ A(x, Y_1, Z_1, Z_2) \quad \checkmark \\
\mathcal{M}_2: & \quad B(U, x, y) \quad \rightarrow \quad \exists Z_1 \exists Z_2 \ A(x, y, Z_1, Z_2)
\end{align*}
\]
We want to define what means to recover sound information

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**Example**

$\mathcal{M}$: $A(x, y, Z, Z) \rightarrow \exists U B(U, x, y)$

$\mathcal{M}_1$: $B(U, x, y) \rightarrow \exists Y_1 \exists Z_1 \exists Z_2 A(x, Y_1, Z_1, Z_2)$ ✓

$\mathcal{M}_2$: $B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 A(x, y, Z_1, Z_2)$ ✓
We want to define what means to recover sound information

- data *may be lost* in the exchange through \( \mathcal{M} \).
- we want an \( \mathcal{M}' \) that *at least* recovers sound data w.r.t. \( \mathcal{M} \).

**Example**

\[
\mathcal{M}: \quad A(x, y, Z, Z) \quad \rightarrow \quad \exists U \; B(U, x, y)
\]

\[
\mathcal{M}_1: \quad B(U, x, y) \quad \rightarrow \quad \exists Y_1 \; \exists Z_1 \; \exists Z_2 \; A(x, Y_1, Z_1, Z_2) \quad \checkmark
\]

\[
\mathcal{M}_2: \quad B(U, x, y) \quad \rightarrow \quad \exists Z_1 \; \exists Z_2 \; A(x, y, Z_1, Z_2) \quad \checkmark
\]

\[
\mathcal{M}_3: \quad B(U, x, y) \quad \rightarrow \quad \exists Z_1 \; \exists Z_2 \; A(x, Z_1, y, Z_2)
\]
We want to define what means to recover sound information data may be lost in the exchange through $\mathcal{M}$. 

we want an $\mathcal{M}'$ that at least recovers sound data w.r.t. $\mathcal{M}$.

Example

$\mathcal{M}$: $A(x, y, Z, Z) \rightarrow \exists U \ B(U, x, y)$

$\mathcal{M}_1$: $B(U, x, y) \rightarrow \exists Y_1 \exists Z_1 \exists Z_2 \ A(x, Y_1, Z_1, Z_2)$  ✓

$\mathcal{M}_2$: $B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 \ A(x, y, Z_1, Z_2)$  ✓

$\mathcal{M}_3$: $B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 \ A(x, Z_1, y, Z_2)$  ✗
We want to define what means to recover sound information.

- data may be lost in the exchange through $\mathcal{M}$.
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Example

\[
\begin{align*}
\mathcal{M}: & \quad A(x, y, Z, Z) \rightarrow \exists U \ B(U, x, y) \\
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\mathcal{M}_2: & \quad B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 \ A(x, y, Z_1, Z_2) \quad \checkmark \\
\mathcal{M}_3: & \quad B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 \ A(x, Z_1, y, Z_2) \quad \times
\end{align*}
\]

We call mapping $\mathcal{M}_1$ and $\mathcal{M}_2$ recoveries of $\mathcal{M}$. 
A mapping is a recovery of $\mathcal{M}$ if it recovers sound information wrt to $\mathcal{M}$.
A mapping is a *recovery* of $\mathcal{M}$ if it recovers sound information wrt to $\mathcal{M}$

**Definition**

A mapping $\mathcal{M}'$ is a *recovery* of $\mathcal{M}$ iff for every source instance $I$:

$$(I, I) \in \mathcal{M} \circ \mathcal{M}'$$
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Definition

A mapping $\mathcal{M}'$ is a recovery of $\mathcal{M}$ iff for every source instance $I$:

$$(I, I) \in \mathcal{M} \circ \mathcal{M}'$$
We want to choose the most informative recovery of $\mathcal{M}$
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- Can we compare alternatives recoveries?
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Example

$\mathcal{M}: \ A(x, y, Z, Z) \rightarrow \exists U \ B (U, x, y)$
We want to choose the most informative recovery of $\mathcal{M}$

- Can we compare alternatives recoveries?

**Example**

$\mathcal{M}$: $A(x, y, Z, Z) \rightarrow \exists U \ B(U, x, y)$

$\mathcal{M}_1$: $B(U, x, y) \rightarrow \exists Y_1 \ \exists Z_1 \ \exists Z_2 \ A(x, Y_1, Z_1, Z_2)$

$\mathcal{M}_2$: $B(U, x, y) \rightarrow \exists Z_1 \ \exists Z_2 \ A(x, y, Z_1, Z_2)$
We want to choose the most informative recovery of $\mathcal{M}$

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$\mathcal{M}$: $A(x, y, Z, Z) \rightarrow \exists U \ B(U, x, y)$

$\mathcal{M}_1$: $B(U, x, y) \rightarrow \exists Y_1 \exists Z_1 \exists Z_2 \ A(x, Y_1, Z_1, Z_2)$

$\mathcal{M}_2$: $B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 \ A(x, y, Z_1, Z_2)$

$\mathcal{M}_2$ is better than $\mathcal{M}_1$
We want to choose the most informative recovery of $\mathcal{M}$

- Can we compare alternatives recoveries?

**Example**

$\mathcal{M}$: $A(x, y, Z, Z) \rightarrow \exists U \; B(U, x, y)$

$\mathcal{M}_1$: $B(U, x, y) \rightarrow \exists Y_1 \exists Z_1 \exists Z_2 \; A(x, Y_1, Z_1, Z_2)$

$\mathcal{M}_2$: $B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 \; A(x, y, Z_1, Z_2)$

$\mathcal{M}_4$: $B(U, x, y) \rightarrow \exists Z \; A(x, y, Z, Z)$

$\mathcal{M}_2$ is better than $\mathcal{M}_1$
We want to choose the most informative recovery of $\mathcal{M}$

- Can we compare alternatives recoveries?

Example

$\mathcal{M}$: $A(x, y, Z, Z) \rightarrow \exists U \ B(U, x, y)$

$\mathcal{M}_1$: $B(U, x, y) \rightarrow \exists Y_1 \exists Z_1 \exists Z_2 \ A(x, Y_1, Z_1, Z_2)$

$\mathcal{M}_2$: $B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 \ A(x, y, Z_1, Z_2)$

$\mathcal{M}_4$: $B(U, x, y) \rightarrow \exists Z \ A(x, y, Z, Z)$

$\mathcal{M}_2$ is better than $\mathcal{M}_1$

$\mathcal{M}_4$ is better than $\mathcal{M}_2$ and $\mathcal{M}_1$
We want to choose the most informative recovery of $\mathcal{M}$

- Can we compare alternatives recoveries?

Example

$\mathcal{M}$: $A(x, y, Z, Z) \rightarrow \exists U \ B(U, x, y)$

$\mathcal{M}_1$: $B(U, x, y) \rightarrow \exists Y_1 \exists Z_1 \exists Z_2 \ A(x, Y_1, Z_1, Z_2)$

$\mathcal{M}_2$: $B(U, x, y) \rightarrow \exists Z_1 \exists Z_2 \ A(x, y, Z_1, Z_2)$

$\mathcal{M}_4$: $B(U, x, y) \rightarrow \exists Z \ A(x, y, Z, Z)$

$\mathcal{M}_2$ is better than $\mathcal{M}_1$

$\mathcal{M}_4$ is better than $\mathcal{M}_2$ and $\mathcal{M}_1$

We call mapping $\mathcal{M}_4$ a maximum recovery of $\mathcal{M}$. 
A mapping is a *maximum recovery* of $\mathcal{M}$ if it is the *smallest* recovery of $\mathcal{M}$.
A mapping is a \textit{maximum recovery} of $\mathcal{M}$ if it is the \textit{smallest} recovery of $\mathcal{M}$

Definition

Given a $\mathcal{M}'$ and $\mathcal{M}''$ recoveries of $\mathcal{M}$, we say that $\mathcal{M}'$ is \textit{at least as informative} as $\mathcal{M}''$ for $\mathcal{M}$ or
A mapping is a maximum recovery of $\mathcal{M}$ if it is the smallest recovery of $\mathcal{M}$

Definition

Given a $\mathcal{M}'$ and $\mathcal{M}''$ recoveries of $\mathcal{M}$, we say that $\mathcal{M}'$ is at least as informative as $\mathcal{M}''$ for $\mathcal{M}$ or

$$\mathcal{M}'' \preceq \mathcal{M}'$$ if and only if $\mathcal{M} \circ \mathcal{M}' \subseteq \mathcal{M} \circ \mathcal{M}''$. 
A mapping is a *maximum recovery of* $\mathcal{M}$ if it is the *smallest* recovery of $\mathcal{M}$

**Definition**

Given a $\mathcal{M}'$ and $\mathcal{M}''$ recoveries of $\mathcal{M}$, we say that $\mathcal{M}'$ is *at least as informative as* $\mathcal{M}''$ for $\mathcal{M}$ or $\mathcal{M}'' \preceq \mathcal{M}'$ if and only if $\mathcal{M} \circ \mathcal{M}' \subseteq \mathcal{M} \circ \mathcal{M}''$. 
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**Definition**

$\mathcal{M}'$ is a *maximum recovery* of $\mathcal{M}$ if $\mathcal{M}'$ is a maximum w.r.t. $\preceq$. 
Maximum recovery strictly generalize previous notions
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In [Fag06], Fagin gives the first definition of an inverse of a mapping:

- Focused on the relational case and st-tgds.
Maximum recovery strictly generalize previous notions

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**Theorem**

Let $\mathcal{M}$ be a mappings specified by st-tgds and fagin-invertible, then:

$\mathcal{M}'$ is an fagin-inverse of $\mathcal{M}$ $\iff$ $\mathcal{M}'$ is a maximum recovery of $\mathcal{M}$. 
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**Theorem**

Let $\mathcal{M}$ be a mappings specified by st-tgds and fagin-invertible, then:

$\mathcal{M}'$ is an fagin-inverse of $\mathcal{M} \iff \mathcal{M}'$ is a maximum recovery of $\mathcal{M}$.

**Advantage of maximum recoveries:**

- Fagin inverses rarely exist for st-tgds.
- Maximum recoveries always exist for st-tgds.
How can we compute maximum recoveries?
Certain answers and query rewriting are the key concepts in the algorithm.
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*Certain answers and query rewriting:*

- are old concepts in data integration.
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Definition

Given a mapping $\mathcal{M}$ and a source instance $I$, we say that $\bar{t}$ is a certain answer for $Q$ over $I$ iff
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We denote by $\text{certain}_{\mathcal{M}}(Q, I)$ the set of certain answers.
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Example

$\mathcal{M}$:

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\begin{align*}
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Q_1(u, v): & \quad B(u, v)
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$Q_{1}(u, v)$: $B(u, v)$

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$Q_2(u): \ \exists Z \ B(u, Z)$

$\text{certain}_{\mathcal{M}}(Q_2, I) = \{1\}$
Query rewriting:
reformulating a target query in the source
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Query rewriting: reformulating a target query in the source

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Given a mapping $\mathcal{M}$ and a target query $Q$, we say that $Q'$ is a source rewriting of $Q$ under $\mathcal{M}$ if

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If $\mathcal{M}$ is specified by st-tgds, then for every target query $Q \in \text{CQ}$, there is a source rewriting $Q' \in \text{UCQ}^=$ of $Q$ under $\mathcal{M}$. 

Computing a maximum recovery using rewriting of queries
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For a mapping \( M \) specified by st-tgds, compute \( M' \) as follows:
Computing a maximum recovery using rewriting of queries

For a mapping $\mathcal{M}$ specified by st-tgds, compute $\mathcal{M}'$ as follows:

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For every dependency $\varphi(\bar{x}) \rightarrow \exists \bar{y} \psi(\bar{x}, \bar{y})$ defining $\mathcal{M}$:
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- Let $\alpha(\bar{x}) \in \text{UCQ}^\mathcal{M}$ be a source rewriting of $\exists \bar{y} \psi(\bar{x}, \bar{y})$ under $\mathcal{M}$.
- Add to the definition of $\mathcal{M}'$ the dependency

$$\exists \bar{y} \psi(\bar{x}, \bar{y}) \land \mathbf{C}(\bar{x}) \rightarrow \alpha(\bar{x}).$$
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\( M' \) is a maximum recovery of \( M \).
Maximum recovery algorithm is quadratic in the full case, exponential in general
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<table>
<thead>
<tr>
<th></th>
<th>full st-tgds</th>
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<tbody>
<tr>
<td>Output:</td>
<td>CQ-to-UCQ(^=)</td>
<td>(\text{CQ}^{c(\cdot)})-to-UCQ(^=)</td>
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Some highlights:
- We use query-rewriting as a black box in the algorithm.
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Some highlights:

- We use query-rewriting as a black box in the algorithm.
- Disjunction and predicate $\text{C}(\cdot)$ is the language needed to specify maximum recoveries for st-tgds.
We need to extend the language of st-tgds in order to express maximum recoveries.
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Problems:

- The algorithm returns a set of $\text{CQ}^C(\cdot)\text{-to-UCQ}^-$ dependencies.
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- st-tgds always have an inverse, and
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- st-tgds always have an inverse, and
- such inverse can be expressed in a language with the same good properties as st-tgds for data exchange.
How can we compute an inverse without disjunctions?
We can parameterize the notion of maximum recovery by a class of queries
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**Maximum recoveries:**

- recover the maximum amount of sound information.
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If we concentrate in conjunctive information as st-tgds, we can find more practical inverses.
We want to recover sound information with respect to a query.
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Example

$\mathcal{M}: \quad A(x, y, z) \longrightarrow \exists U B(x, y, U)$
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\[ M': \quad B(x, y, u) \quad \rightarrow \quad A(x, y, x) \]
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$\mathcal{M}'$: $B(x, y, u) \rightarrow A(x, y, x)$

$I$: \{A(1, 2, 3)\}
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\[ \text{Sol}_{M \circ M'}(I): \ \{ A(1, 2, 1) \}, \ \{ A(1, 2, 1), A(3, 4, 5) \}, \ldots \]
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$Q_1(x)$: $\exists Y \exists Z A(x, Y, Z)$
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We call mapping \( M' \) a \( Q_1 \)-recovery of \( M \).
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We can extend the definition of $Q$-recovery to a class of queries $C$. 
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Definition

Given a class of queries $C$, we say that $M'$ is a $C$-recovery of $M$ if

$$\text{certain}_{M \circ M'}(Q, I) \subseteq Q(I)$$

for every source instance $I$ and for every query $Q \in C$. 
We can extend the definition of $Q$-recovery to a class of queries $\mathcal{C}$

Definition

Given a class of queries $\mathcal{C}$, we say that $\mathcal{M}'$ is a $\mathcal{C}$-recovery of $\mathcal{M}$ if

$$\text{certain}_{\mathcal{M} \circ \mathcal{M}'}(Q, I) \subseteq Q(I)$$

for every source instance $I$ and for every query $Q \in \mathcal{C}$.

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We can extend the definition of $Q$-recovery to a class of queries $C$

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- **All**-recovery: sound information for *all possible queries*.
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We want to recover the maximum amount of information with respect to $C$.
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for every $I$ and $Q \in C$, then

$M_1$ is a **better** than $M_2$ as a $C$-recovery of $M$.

We want a mapping such that the certain answers are **as close as possible** to $Q(I)$. 
A mapping is a $\mathcal{C}$-maximum recovery of $\mathcal{M}$ if it is the best $\mathcal{C}$-recovery of $\mathcal{M}$.
A mapping is a \textit{C-maximum recovery} of \( \mathcal{M} \) if it is the best \( \mathcal{C} \)-recovery of \( \mathcal{M} \)

\begin{definition}
Given a class of queries \( \mathcal{C} \), we say that \( \mathcal{M}_1 \) is a \textit{C-maximum recovery} of \( \mathcal{M} \) if for every \( \mathcal{C} \)-recovery \( \mathcal{M}_2 \) of \( \mathcal{M} \), it holds that
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Given a class of queries $C$, we say that $\mathcal{M}_1$ is a $C$-maximum recovery of $\mathcal{M}$ if for every $C$-recovery $\mathcal{M}_2$ of $\mathcal{M}$, it holds that

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for every source instance $I$ and for every query $Q \in C$. 
A mapping is a $\mathcal{C}$-maximum recovery of $\mathcal{M}$ if it is the best $\mathcal{C}$-recovery of $\mathcal{M}$.

**Definition**

Given a class of queries $\mathcal{C}$, we say that $\mathcal{M}_1$ is a $\mathcal{C}$-maximum recovery of $\mathcal{M}$ if for every $\mathcal{C}$-recovery $\mathcal{M}_2$ of $\mathcal{M}$, it holds that

$$\text{certain}_{\mathcal{M}_2} (Q, I) \subseteq \text{certain}_{\mathcal{M}_1} (Q, I) \subseteq Q(I)$$

for every source instance $I$ and for every query $Q \in \mathcal{C}$.

$\mathcal{M}_1$ is better than any other possible $\mathcal{C}$-recovery!
Previous notions of inverse correspond to particular classes of queries.
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Let $\mathcal{M}$ be specified by st-tgds:
Previous notions of inverse correspond to particular classes of queries

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**Theorem**

- If $\mathcal{M}'$ is a Fagin-inverse of $\mathcal{M}$
  then $\mathcal{M}'$ is a UCQ$\neq$-maximum recovery of $\mathcal{M}$. 
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We can use different classes of queries to find more practical and useful inverses.
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We know that:

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We know that:

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Theorem

Every mapping specified by st-tgds has a CQ-maximum recovery that can be specified by st-tgds with $\neq$ and $C(\cdot)$ in the left-hand side.
How can we compute a CQ-maximum recovery?
Computing a CQ-maximum recovery consists of two main steps.
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Algorithm

- Step 1: Compute a maximum recovery $\mathcal{M}'$ of $\mathcal{M}$. 
Computing a **CQ**-maximum recovery consists of two main steps

**Algorithm**

- **Step 1**: Compute a maximum recovery $\mathcal{M}'$ of $\mathcal{M}$.

- **Step 2**: Eliminate disjunctions and equalities of $\mathcal{M}'$.
  - **Step 2.1**: Eliminate equalities.
  - **Step 2.2**: Eliminate disjunctions.
Step 1: Compute a maximum recovery
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Step 1: Compute a maximum recovery

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**Problem:** disjunctions and equalities in the right-hand side.
Step 2.1: Eliminate right-hand side equalities
Step 2.1: Eliminate right-hand side equalities

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(x, y) \rightarrow B(x, y) \lor (C(x) \land x = y)$</td>
</tr>
</tbody>
</table>
Step 2.1: Eliminate right-hand side equalities

Example

\[ A(x, y) \rightarrow B(x, y) \lor (C(x) \land x = y) \]

\[ \downarrow \]

\[ A(x, y) \land x \neq y \rightarrow B(x, y) \]
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Example

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A(x, y) & \rightarrow B(x, y) \lor (C(x) \land x = y) \\
\downarrow & \\
A(x, y) \land x \neq y & \rightarrow B(x, y) \\
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- Let $\mathcal{M}'$ be the mapping constructed in Step 1.
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\( \mathcal{M}'' \) is a \( \text{CQ} \)-maximum recovery of \( \mathcal{M} \).
Key concepts in Step 2.2
We have the following rule:
\[ \varphi(\bar{x}) \rightarrow \beta_1(\bar{x}) \lor \beta_2(\bar{x}) \lor \cdots \lor \beta_n(\bar{x}) \]
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The key concepts in Step 2.2 are *homomorphisms* and *Cartesian product of queries*. 
Key concepts in Step 2.2: Homomorphisms
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$$\exists u \exists v \ A(x_1, u) \land B(v, v) \quad \xrightarrow{h} \quad A(x_1, x_2) \land B(x_1, x_1)$$
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**Example**

$$\exists u \exists v \ A(x_1, u) \land B(v, v) \xrightarrow{h} A(x_1, x_2) \land B(x_1, x_1)$$

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- $h(v) = x_1$
- $h(x_1) = x_1$
Key concepts in Step 2.2:
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Definition (semantic version)

The conjunctive query $Q$ is the *Cartesian product* of $Q_1$ and $Q_2$, or

$$Q_1 \times Q_2$$

if it is the *closest query* to both $Q_1$ and $Q_2$ in terms of homomorphism.
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\[
\begin{array}{c}
Q_1 \\
\downarrow h_1 \\
Q \\
\downarrow h_1' \\
Q' \\
\downarrow h_2' \\
Q_2 \\
\downarrow h_2 \\
\end{array}
\]
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A simple extension of the Cartesian product of graphs.
Step 2.2: Eliminate disjunctions
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Step 2.2: Eliminate disjunctions

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- Let $M''$ be the mapping constructed in Step 2.1.
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$$\varphi(\bar{x}) \rightarrow \beta_1(\bar{x}) \lor \beta_2(\bar{x}) \lor \cdots \lor \beta_n(\bar{x})$$
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\[
\varphi(\bar{x}) \quad \rightarrow \quad \beta_1(\bar{x}) \lor \beta_2(\bar{x}) \lor \cdots \lor \beta_n(\bar{x})
\]

\[
\downarrow
\]

\[
\varphi(\bar{x}) \quad \rightarrow \quad \beta_1(\bar{x}) \times \beta_2(\bar{x}) \times \cdots \times \beta_n(\bar{x})
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  \]
  \[
  \text{⇓}
  \]
  \[
  \varphi(\bar{x}) \quad \rightarrow \quad \beta_1(\bar{x}) \times \beta_2(\bar{x}) \times \cdots \times \beta_n(\bar{x})
  \]
- $\mathcal{M}^*$ is a set of $\text{CQ}^c(\cdot), \neq$-to-$\text{CQ}$ dependencies.
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\begin{align*}
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\downarrow & \\
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- $\mathcal{M}^*$ is a set of $\text{CQ}^c(\cdot),\neq$-to-$\text{CQ}$ dependencies.

$\mathcal{M}^*$ is a $\text{CQ}$-maximum recovery of $\mathcal{M}$.
Summing up...

Algorithm

- Step 1: Compute a maximum recovery $\mathcal{M}'$ of $\mathcal{M}$.

- Step 2: Eliminate disjunctions and equalities of $\mathcal{M}'$.
  - Step 2.1: Eliminate equalities using inequalities in the left side.
  - Step 2.2: Eliminate disjunctions using Cartesian product of queries.
Algorithm

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  - Step 2.1: Eliminate equalities using inequalities in the left side.
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The output is a **CQ**-maximum recovery of $M$ specified by $\text{st-tgds} \neq, c(\cdot)$. 

Summing up...
st-tgds\neq,^C is the right language to specify an inverse
st-tgds$\neq$,\(C\) is the right language to specify an inverse

Theorem

The language of st-tgds$\neq$,\(C\) is the minimal language needed to specify \(CQ\)-maximum recoveries of st-tgds.
Theorem

The language of $\mathsf{st-tgds}^{\neq, \mathcal{C}}$ is the \textit{minimal language} needed to specify $\mathsf{CQ}$-maximum recoveries of $\mathsf{st-tgds}$.

This language has the same good properties as $\mathsf{st-tgds}$, in particular:

- the \textit{chase} procedure can be used to exchange data,
The language of $\text{st-tgds}^{\neq,c}$ is the \textit{minimal language} needed to specify \textbf{CQ}-maximum recoveries of st-tgds.

This language has the same good properties as st-tgds, in particular:

- the \textit{chase} procedure can be used to exchange data,
- a \textit{canonical universal solution} exists for every source instance.
Review
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- *Maximum recovery*: recovers the maximum amount of sound information.
Review

- **Recovery**: recovers sound information.

- **Maximum recovery**: recovers the maximum amount of sound information.

- **Query rewriting** plays a key role to compute maximum recoveries.
Review

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- **$C$-recovery**: recovers sound information wrt a class of queries $C$. 
Review


- *Maximum recovery*: recovers the maximum amount of sound information.

- *Query rewriting* plays a key role to compute maximum recoveries.

- *$C$-recovery*: recovers sound information wrt a class of queries $C$.

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Review

- **Recovery**: recovers sound information.

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- **Query rewriting** plays a key role to compute maximum recoveries.

- **C-recovery**: recovers sound information wrt a class of queries $C$.

- **C-maximum recovery**: recovers the maximum amount of information wrt a class of queries $C$.

- **Cartesian product of queries** plays a key role to compute $C$-maximum recoveries.
Conclusion

- *Maximum recovery* is the *best* that we can do to invert mappings.
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- *Certain answers* and *query rewriting* are old concepts that naturally arise when we study the inverse problem.
Conclusion

- Maximum recovery is the best that we can do to invert mappings.

- Certain answers and query rewriting are old concepts that naturally arise when we study the inverse problem.

- If we focus in certain kind of queries we can find more practical solutions to recovering information in data exchange.
Recovering information in data exchange

Cristian Riveros
Khipu Institute

Oxford University
Tue 26 Jan 2010